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LETTER TO THE EDITOR

Transfer matrix calculation of the relative noise exponent in a two-dimensional percolating network

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Abstract. A new method is given for calculating the relative noise exponent describing the size dependence of macroscopic fluctuations due to microscopic resistance noise of a random resistor network at the percolation threshold p_c. The macroscopic fluctuation is calculated exactly using the transfer matrix idea in a given network. The asymptotic exponent b is found by finite-size scaling.

The critical behaviour of random networks has motivated much recent work. Quantities such as the fractal dimension [1], correlation length exponent, spectral dimension [2, 3] and spreading dimension [4, 5] are used to describe the geometrical properties of percolation clusters. However, these quantities are not sufficient for a characterisation of all the physical properties of a self-similar structure. Calculations on different fractals [6-8] and ε expansion [9] indicated that an infinite number of exponents describe resistance fluctuations arising from the microscopic noise of individual resistors of the network. Park *et al* [9] have suggested the existence of an extended family of exponents. From the work of de Arcangelis *et al* [7] one knows that an anomalous voltage distribution of random networks is the common origin of the existence of this family of exponents.

The purpose of the present letter is to present a method for measuring the relative noise exponent b. The basic quantity in which we are interested is the magnitude of the relative noise $\mathscr{S}_R = \{\langle \delta R \delta R \rangle / R^2\}$ where R and δR are, respectively, the overall resistance and its fluctuation. $\langle\ \rangle$ means an average over the noise for one network; $\{\ \}$ an average over different networks. For a strip of width n and length L, with $L \to \infty$ at fixed n, it varies at p_c asymptotically as

$$\lim_{L\to\infty} L\mathscr{S}_R \sim n^{-b-1}.$$

This exponent, proposed by Rammal *et al* [8], is closely related to the first few members of the exponent family of resistance noise. The method is very similar to the transfer matrix method for calculating the conductivity and resistivity exponent [10]. The main advantage is that it gives the correlation function of the resistivity matrix exactly. But to do this one has to apply a matrix with four indices. This is a certain disadvantage of the method. We compute the resistivity and the resistivity-resistivity correlation per unit length of networks consisting of long strips with resistors placed randomly on a square lattice. The results obtained for varying strip widths are analysed with

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the help of finite-size scaling. They give an estimate for the two-dimensional relative noise exponent b.

The simplest version of the model can be formulated as follows. There are two kinds of resistors: superconducting with probability $p_c = \frac{1}{2}$ and normal conducting with probability $1 - p_c$ and only the normal resistances fluctuate independently with the same frequency spectrum. We note that in previous work [8, 9] a mixture of normal conducting and insulating bonds had been considered instead. The strip is constructed by adding bond after bond. In the transverse direction we impose periodic boundary conditions which have been used in the case of random mixtures of normal and superconducting resistors [11]. In the first vertical plane the voltage is zero. If the configuration of currents attached to the last plane is $I_1, \ldots, I_n$ then the voltages on this plane are given by the resistivity matrix:

$$U_i = \sum_{j=1}^{n} R_{ij} I_j \qquad \text{for } i = 1\text{-}n.$$

For each bond that one adds, R_{ij} has to be transformed [11].

Now let us assume that we have constructed the strip in this way and that we know the resistivity matrix R. If the resistance of the normal resistors is not fixed but fluctuates around the average value, the change of R_{ij} is given by the first term of a weak-disorder expansion of R_{ij}:

$$\delta R_{ij} = \sum_q \frac{\partial R_{ij}}{\partial r_q} \delta r_q$$

where the summation is taken over all normal resistors. The derivatives $\partial R_{ij}/\partial r_q$ depend only on the unperturbed network. We can calculate correlation functions such as

$$C_{ijkl} = \langle \delta R_{ij} \delta R_{kl} \rangle.$$

After adding a new bond C_{ijkl} has to be transformed. Thus one can obtain recursion formulae for C_{ijkl}. For a very long strip, the overall resistance and resistance fluctuation per unit length are given by

$$\rho_n = \lim_{L\to\infty} (R_{ii}/L) \qquad c_n = \langle \delta R \delta R \rangle = \lim_{L\to\infty} (C_{iiii}/L)$$

independently of the endpoint i.

Suppose that the behaviour of c_n in the large n limit is

$$nc_n \sim n^{-\varepsilon} \qquad (p = p_c)$$

then, since

$$n\rho_n \sim n^{-s/\nu} \qquad (p = p_c)$$

we have $b = \varepsilon - 2s/\nu$. In two dimensions, because of exact duality [12], b is the same for a normal insulator and a normal superconducting mixture; $p_c = \frac{1}{2}$ in the two cases. Thus from these results we know also the exponent x_2 proposed by Rammal *et al* [8].

The results of our calculation obtained by using about 5 h on a Prime computer are displayed in figure 1. Our result is

$$b = 1.2$$

with the error bar about 10%, b being somewhat greater than previously published. To compute the exponent b more precisely we must choose a greater length. We hope

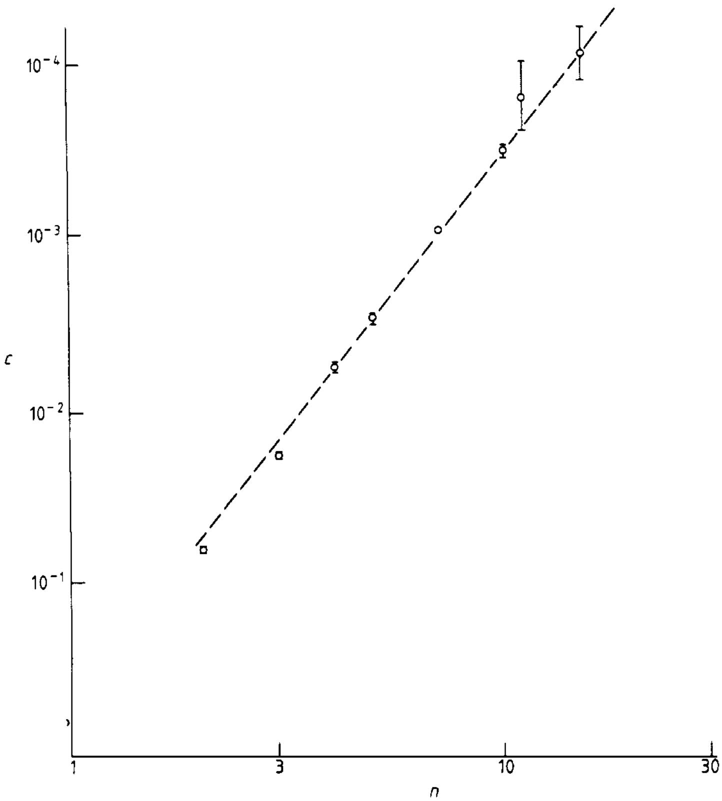

Figure 1. Log-log plot of c_n against the strip width n. Error bars arise from the statistical errors. $L=1000$ for $n=2, 3, 4, 5$, $L=5000$ for $n=7, 10, 11$ and $L=2000$ for $n=15$.

that we will be able to repeat our calculation on a Cray vector computer. We have already succeeded in vectorising the program [13].

In summary, we have proposed a new method based on the transfer matrix approach. For our model in two dimensions the relative noise has the same critical behaviour as for a normal insulator mixture. Our method is not limited to any spatial dimension and we hope that we will also be able to perform calculations in three dimensions.

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